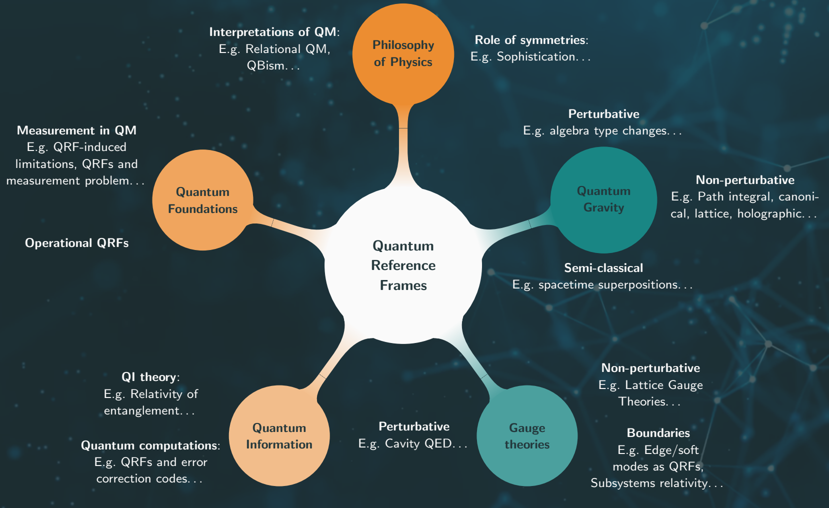
Quantum Reference Frames for Quantum Gravity: The Relational Gravitational Path Integral

Mostly based on 2607.21463, but also review of past literature

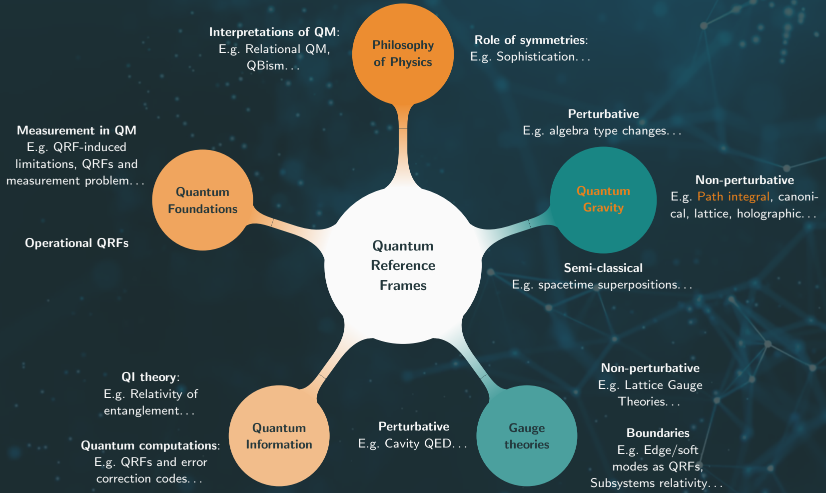
Luca Marchetti

Quantum Crossroads
Kavli IPMU
August 7, 2026

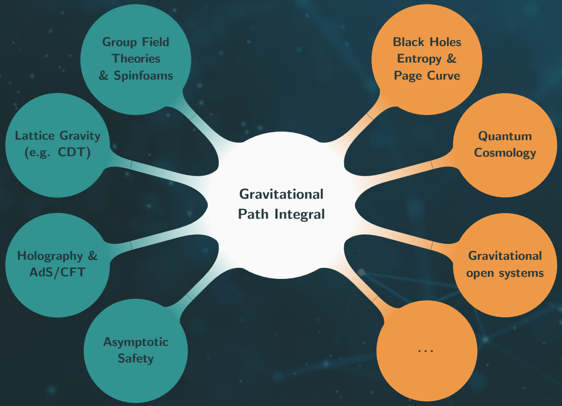
OIST
Kavli IPMU

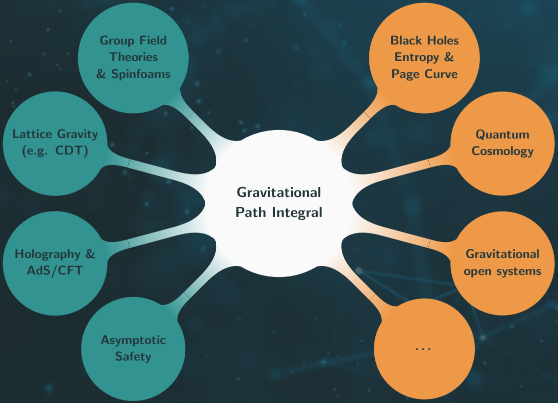


QRFs are a toolkit for quantization in the presence of symmetry



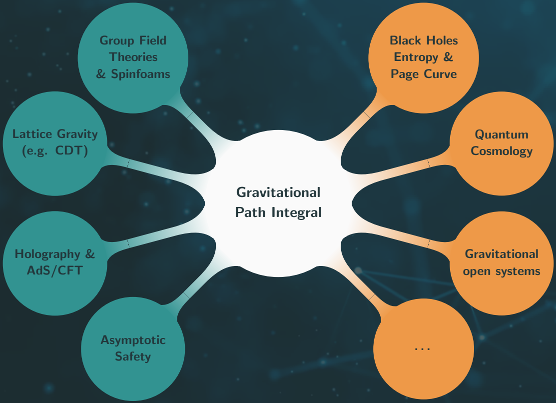
QRFs are a toolkit for quantization in the presence of symmetry


Group Field Theories & Spinfoams
Lattice Gravity (e.g. CDT)
Holography & AdS/CFT
Asymptotic Safety
Gravitational Path Integral
Black Holes Entropy & Page Curve
Quantum Cosmology
Gravitational open systems
...



- How to regularize without breaking diffeos?
- Transition amplitudes and constraints?
- Gauge-invariant local correlators?
- Which generating functionals?
- Which ground states?
- ...

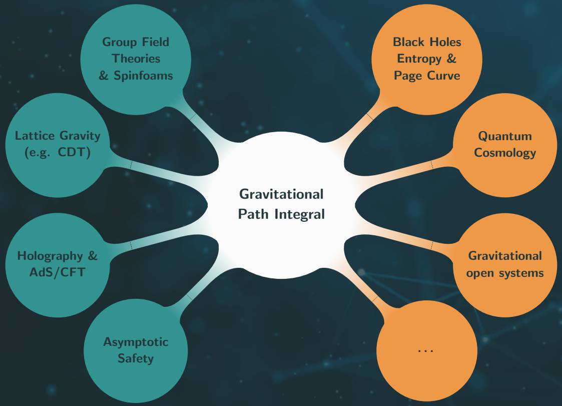
Gravitational PI $\sim \delta(\hat{C})$



- How to regularize without breaking diffeos?
- Transition amplitudes and constraints?
- Gauge-invariant local correlators?
- Which generating functionals?
- Which ground states?
- . . .

Background Indep.

Gravitational PI $\sim \delta(\hat{C})$



- How to regularize without breaking diffeos?
- Transition amplitudes and constraints?
- Gauge-invariant local correlators?
- Which generating functionals?
- Which ground states?
- ...

Background Indep.

Formal answers using QRFs!

Part I

QRF and Mechanical Models

- What are QRF? How do things look like from different QRFs perspectives?
- How can we quantize a mechanical model with gauge symmetry using QRFs?
- Can we extract a general procedure to use in gravity?

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- What are QRF? How do things look like from different QRFs perspectives?
- How can we quantize a mechanical model with gauge symmetry using QRFs?
- Can we extract a general procedure to use in gravity?

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

QRF for a mechanical model

$$L[\dot{q}_i, q_i] = K[\dot{q}_i] + V(q_i)$$

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{q}_i, q_i] = \sum_{i=1}^{N} \dot{q}_i^2 - N^{-1}\Big(\sum_{i=1}^{N} \dot{q}_i\Big)^2 + V(\{q_i - q_j\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{q}_i, q_i] = \sum_{i=1}^{N} \dot{q}_i^2 - N^{-1}\Big(\sum_{i=1}^{N} \dot{q}_i\Big)^2 + V(\{q_i - q_j\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint $=$ redundant information!

Relational description[1]

- Gauge-inv. obs. $=$ relative distances: $Q_{i|1} \equiv q_i - q_1$.
- $q_1 =$ dynamical frame: behaves as dyn. "coordinate".

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{q}_i, q_i] = \sum_{i=1}^{N} \dot{q}_i^2 - N^{-1}\Big(\sum_{i=1}^{N} \dot{q}_i\Big)^2 + V(\{q_i - q_j\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\rm tot} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv q_i - q_2$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

$$L[\dot{q}_i, q_i] = \sum_{i=1}^{N} \dot{q}_i^2 - N^{-1}\Big(\sum_{i=1}^{N} \dot{q}_i\Big)^2 + V(\{q_i - q_j\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint $=$ redundant information!

Relational description[1]

- Gauge-inv. obs. $=$ relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i\neq j} \dot{Q}_{i|j}^2 - N^{-1}\Big(\sum_{i\neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i\neq j} \dot{Q}^2_{i|j} - N^{-1}\Big(\sum_{i\neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint $=$ redundant information!

Relational description[1]

- Gauge-inv. obs. $=$ relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$\tilde{Z} = \int \prod_i \mathcal{D}q_i \, e^{iS[q_i]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i\neq j} \dot{Q}_{i|j}^2 - N^{-1}\Big(\sum_{i\neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$\tilde{Z} = \int \mathcal{D}q_j \int \mathcal{D}Q_{i|j} e^{iS[Q_{i|j}]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i \neq j} \dot{Q}_{i|j}^2 - N^{-1} \Big(\sum_{i \neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \cancel{\int \mathcal{D} q_j} \int \mathcal{D} Q_{i|j} e^{iS[Q_{i|j}]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i \neq j} \dot{Q}_{i|j}^2 - N^{-1} \Big(\sum_{i \neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \int \mathcal{D} Q_{i|j} \, e^{iS[Q_{i|j}]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!
- Gauge-invariant path integral!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i\neq j} \dot{Q}^2_{i|j} - N^{-1}\Big(\sum_{i\neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \int \mathcal{D}Q_{i|k}\, e^{iS[Q_{i|k}]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!
- Gauge-invariant and Perspective-Neutral path integral!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i \neq j} \dot{Q}^2_{i|j} - N^{-1} \Big(\sum_{i \neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint $=$ redundant information!

Relational description[1]

- Gauge-inv. obs. $=$ relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \int \mathcal{D}q_j \delta(q_j - \bar{q}_j) \int \mathcal{D}Q_{i|k} e^{iS[Q_{i|k}]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!
- Gauge-invariant and Perspective-Neutral path integral!
- Equivalent to quantum gauge-fixed (Faddeev-Popov) path integal.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i \neq j} \dot{Q}_{i|j}^2 - N^{-1} \Big(\sum_{i \neq j} \dot{Q}_{i|j}\Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\text{tot}} = 0$.
- Constraint $=$ redundant information!

Relational description[1]

- Gauge-inv. obs. $=$ relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \int \prod_i \mathcal{D}q_i \delta(q_j - \bar{q}_j) e^{iS[q_i]}$$

- The integral over gauge directions parameterized by the quantum reference frame is redundant!
- Gauge-invariant and Perspective-Neutral path integral!
- Equivalent to quantum gauge-fixed (Faddeev-Popov) path integal.

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

QRF for a mechanical model

$$L[\dot{Q}_{i|j}, Q_{i|j}] = \sum_{i \neq j} \dot{Q}_{i|j}^2 - N^{-1} \Big(\sum_{i \neq j} \dot{Q}_{i|j} \Big)^2 + V(\{Q_{i|j}\})$$

Gauge symmetry[1]

- Invariance under $q_i \to q_i + x(t)$: $\mathbf{P}_{\mathrm{tot}} = 0$.
- Constraint = redundant information!

Relational description[1]

- Gauge-inv. obs. = relative distances: $Q_{i|1} \equiv q_i - q_1$.
- Any particle can be dynamical frame: $Q_{i|2} \equiv Q_{i|1} - Q_{2|1}$.

A dynamical frame is a dynamical quantity whose values distinguish gauge-related descriptions.

Quantization

The Perspective Neutral Relational Path-Integral[2]

$$Z = \int \mathcal{D} Q_{i|j} \, e^{iS[Q_{i|j}]}$$

A relational QRF description amounts to a coordinate change in the path integral!

[1] Vanrietvelde, Höhn, Giacomini, Castro-Ruiz 1809.00556; [2] Höhn, Knapper, LM (to appear)

Part I

QRF and Mechanical Models

- What are QRF?
- How can we quantize a mechanical model with gauge symmetry using QRFs?
- Can we extract a general procedure to use in gravity?

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that distinguish gauge-related descriptions.
- How can we quantize a mechanical model with gauge symmetry using QRFs?
- Can we extract a general procedure to use in gravity?

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that distinguish gauge-related descriptions.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- Can we extract a general procedure to use in gravity?

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that distinguish gauge-related descriptions.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with adapted history coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that distinguish gauge-related descriptions.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with adapted history coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Covariance and Invariance[1]

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-covariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, f_\star \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star\varphi, f_\star\mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star\varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory “live” on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star \varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star \varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Physical observables** must be φ-dep. and gauge-invariant: $O[\varphi] = O[f_\star \varphi]$.

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance} : \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star \varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Physical observables** must be φ-dep. and gauge-invariant: $O[\varphi] = O[f_\star \varphi]$.

If $O[\varphi]$ is local $\quad \underbrace{O[\varphi]}_{\text{Depends on } \varphi_{\mathcal{U}}} \quad \underbrace{O[f_\star \varphi]}_{\text{Depends on } \varphi_{f^{-1}(\mathcal{U})}}$

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0 \,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star \varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Physical observables** must be φ-dep. and gauge-invariant: $O[\varphi] = O[f_\star \varphi]$.

$$\text{If } O[\varphi] \text{ is local} \qquad \underbrace{O[\varphi]}_{\text{Depends on } \varphi_{\mathcal{U}}} = \underbrace{O[f_\star \varphi]}_{\text{Depends on } \varphi_{f^{-1}(\mathcal{U})}} \quad \longrightarrow \quad O[\varphi] = O$$

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star \varphi, f_\star \mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star \varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance $+$ background independence $=$ $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star \varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Physical observables** must be φ-dep. and gauge-invariant: $O[\varphi] = O[f_\star \varphi]$, but cannot be $\mathcal{M}$-local.

No spacetime local observables in gravity!

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Invariance and Locality in Classical Gravity

Gravity

Covariance and Invariance[1]

- Theory defined by fields φ and background structures $\mathcal{B}$ on a manifold $\mathcal{M}$, with eom $E[\varphi, \mathcal{B}] = 0$.
- $(\varphi, \mathcal{B})$ support action of diffeos: $(\varphi, \mathcal{B}) \mapsto (f_\star\varphi, f_\star\mathcal{B})$, $f \in \mathrm{Diff}(\mathcal{M})$.

$$\mathrm{Diff}(\mathcal{M})\text{-invariance}: \quad E[\varphi, \mathcal{B}] = 0 \longleftrightarrow E[f_\star\varphi, \mathcal{B}] = 0\,.$$

- $\mathrm{Diff}(\mathcal{M})$-covariance: let the theory "live" on $\mathcal{M}$: no physics, choice of mathematical language.
- $\mathrm{Diff}(\mathcal{M})$-invariance is physical: invariance under active diffemorphisms.
- Gravity: $\mathrm{Diff}(\mathcal{M})$-covariance + background independence = $\mathrm{Diff}(\mathcal{M})$-invariance.

Observables

Gauge-Invariance vs Locality[2]

- **Gauge-symmetries in gravity**: (small) active diffeos: $\varphi \sim f_\star\varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Physical observables** must be φ-dep. and gauge-invariant: $O[\varphi] = O[f_\star\varphi]$, but cannot be $\mathcal{M}$-local.

Should we give up locality?

[1] Giulini 0603087; [2] Goeller, Höhn, Kirklin 2022.

Dynamical Frames and Relational Observables

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \underbrace{\mathcal{U}_R}_{\varphi\text{-dep region on } \mathcal{M}} \to \underbrace{\mathcal{U}_O}_{\text{Region in relational spacetime } \mathcal{O}_R}$ s.t. $R[f_\star \varphi] = R \circ f^{-1}$.

Dynamical Frames and Relational Observables

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star \varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star) A[\varphi]$.

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Dynamical Frames and Relational Observables

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \dots$

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \ldots$

From spacetime locality to relational locality!

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Dynamical Frames and Relational Observables

Frames

Relational Observables

- Dynamical Frames for Diffeos:[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \ldots$

From spacetime locality to relational locality!

Frames

Examples: Scalar fields[3]

- Consider a set of four scalar fields, $Z^\alpha(x) : \mathcal{M} \to \mathbb{R}^4$, $\alpha = 1, \ldots 4$, with $\det Z^\alpha_{,\mu} \neq 0$ globally.

$$\int_{\mathcal{M}} \mathrm{d}x \delta(Z^\alpha_0 - Z^\alpha(x))[\det Z^\alpha_{,\mu}](x) A[\varphi](x) = A[\varphi](Z^{-1}(Z_0)) = Z_\star A[\varphi](Z_0) = O_{A|Z}[\varphi](Z_0)$$

- Inflaton clock: $T[\phi](x) = \bar{\phi}^{-1}(\phi(x)) = t + \pi(x)$, π "Stückelberg field", $\bar{\phi}$ background inflaton.
- Relational observables = gauge-invariant extensions of unitary gauge-fixed quantities.

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Dynamical Frames and Relational Observables

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \ldots$

From spacetime locality to relational locality!

Frames

Examples: Brown-Kuchař dust[3]

$$S_{\text{dust}}[\varphi] = -\frac{1}{2}\int \mathrm{d}x \sqrt{|g|} M(x)(g^{\mu\nu}U_\mu U_\nu + 1)\,, \qquad U_\mu = -\partial_\mu T + \sum_{k=1}^{3} W_k \partial_\mu Z^k\,.$$

- Where $M(x) \neq 0$ flow lines are geodesics (on-shell), labelled by Z^k ($U^\mu \partial_\mu Z^k = 0$).
- Since $U^\mu \partial_\mu T = 1$, T measures the proper time along the geodesics.
- $R[\varphi] = (T(x), Z^k(x))$ maps $M(x) \neq 0$-regions to $\mathcal{O}_R$ = dust space (space of flow lines).

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Dynamical Frames and Relational Observables

Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \ldots$

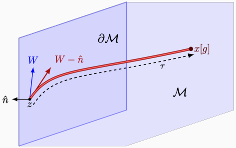
From spacetime locality to relational locality!

Frames

Examples: Geodesic frames[1]

Define points by shooting geodesics from $\partial\mathcal{M}$

- Fix a point $z \in \partial\mathcal{M}$, a vector $W \in \mathrm{T}_z\partial\mathcal{M}$, and $\partial\mathcal{M}$-normal $\hat{n}^a$.
- $x_{\tau,z,W}[g]$ point at distance τ along geodesic def. by z and $W - \hat{n}$.
- $x_{\tau,z,W}[f_\star g] = f_\star x_{\tau,z,W}[g]$ since $\hat{n}$ and geod. eq. are covariant.



Frames

Relational Observables

- **Dynamical Frames for Diffeos:**[1] φ-dependent diffeo $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_O$ s.t. $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-covariant $A[\varphi] \longrightarrow \mathcal{O}_R$-invariant $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

- ✓ **Gauge-Invariant**: Covariance $A[f_\star\varphi] = f_\star A[\varphi]$ implies $O_{A|R}[\varphi] \to (R[\varphi]_\star f^* f_\star A[\varphi]) = O_{A|R}[\varphi]$.
- ✓ **$\mathcal{O}_R$-locality**: If $A[\varphi]$ is local on $\mathcal{U} \subset \mathcal{M}$ ($\mathcal{U} \cap \mathcal{U}_R \neq \emptyset$), then $O_{A|R}[\varphi]$ local on $\mathcal{O}_R$.
- ✓ **E.g.**:[3] Relational metric $O_{g|R}$, relational Lagrangian $O_{L|R} \equiv L_R \ldots$

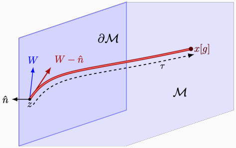
From spacetime locality to relational locality!

Frames

Examples: Geodesic frames[1]

Geometric frame $R^{-1}[g] : (\tau, (z, W)) \mapsto x_{\tau,z,W}[g]$

- Fix a point $z \in \partial\mathcal{M}$, a vector $W \in \mathrm{T}_z\partial\mathcal{M}$, and $\partial\mathcal{M}$-normal $\hat{n}^a$.
- $x_{\tau,z,W}[g]$ point at distance τ along geodesic def. by z and $W - \hat{n}$.
- $x_{\tau,z,W}[f_\star g] = f_\star x_{\tau,z,W}[g]$ since $\hat{n}$ and geod. eq. are covariant.



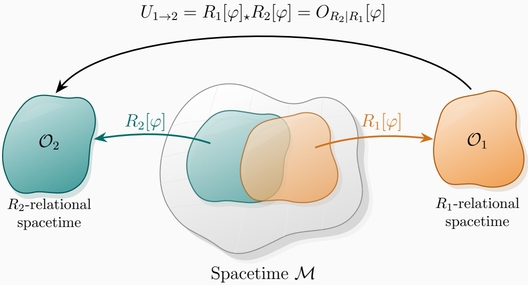
[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Frame Changes and Frame Reorientations

Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]_\star O_{A|R_1}[\varphi] = (R_2[\varphi]_\star R_1[\varphi]^\star R_1[\varphi]_\star A[\varphi]) = R_2[\varphi]_\star A[\varphi] = O_{A|R_2}[\varphi]$
- **E.g.** (Mechanical model): $U_{i\to j} = q_i - q_j$ changes observables, but particles positions are fixed!



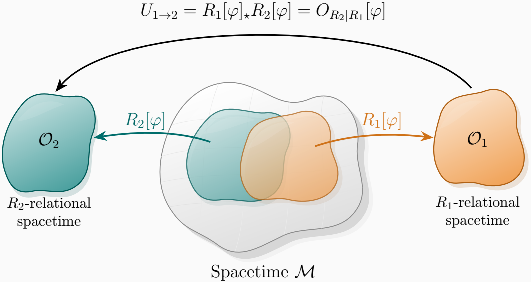
Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

> Frame covariance: Frame changes change the observables, but not the field configuration.

- **E.g.** (Mechanical model): $U_{i\to j} = q_i - q_j$ changes observables, but particles positions are fixed!



[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

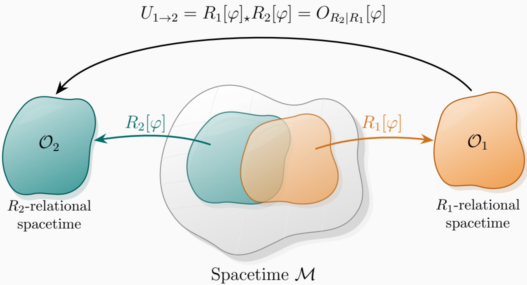
Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

> Frame covariance: Frame changes change the observables, but not the field configuration.

- **Relational atlas**: Frames are $(\mathcal{M}, \mathcal{F})$-local. Frame changes define transition maps for meta-atlas!



[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

> Frame covariance: Frame changes change the observables, but not the field configuration.

- **Relational atlas**: Frames are $(\mathcal{M}, \mathcal{F})$-local. Frame changes define transition maps for meta-atlas!

Symmetries

Frame Reorientations

- **Reorientations**:[1] Change configuration of $R[\varphi]$ only: φ-indep. $R[\varphi] \to \tilde{f} \circ R[\varphi]$, $\tilde{f} \in \mathrm{Diff}(\mathcal{O}_R)$.

> Physical transformations: Reorientations change the observables by changing the R-configuration.

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Frame Changes and Frame Reorientations

Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

> Frame covariance: Frame changes change the observables, but not the field configuration.

- **Relational atlas**: Frames are $(\mathcal{M}, \mathcal{F})$-local. Frame changes define transition maps for meta-atlas!

Symmetries

Frame Reorientations

- **Reorientations:**[1] Change configuration of $R[\varphi]$ only: φ-indep. $R[\varphi] \to \tilde{f} \circ R[\varphi]$, $\tilde{f} \in \mathrm{Diff}(\mathcal{O}_R)$.

> Physical transformations: Reorientations change the observables by changing the R-configuration.

✓ **E.g.** (mechanical model): Translate frame $q_j \to q_j + \bar{q}_j$. Then $O_{i|j} \to O_{i|j}(\bar{q}_j) = O_{i|j} - \bar{q}_j$.

- If they map solutions into solutions (i.e., $L_R \equiv R_\star L$ is covariant), they are physical symmetries.

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Frame Changes and Frame Reorientations

Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

Frame covariance: Frame changes change the observables, but not the field configuration.

- **Relational atlas**: Frames are $(\mathcal{M}, \mathcal{F})$-local. Frame changes define transition maps for meta-atlas!

Symmetries

Frame Reorientations

- **Reorientations:**[1] Change configuration of $R[\varphi]$ only: φ-indep. $R[\varphi] \to \tilde{f} \circ R[\varphi]$, $\tilde{f} \in \mathrm{Diff}(\mathcal{O}_R)$.

Physical transformations: Reorientations change the observables by changing the R-configuration.

Time evolution

- Reorientations generated by φ-indep. timelike $\rho_R \in \mathfrak{X}(\mathcal{O}_R)$, and $L_R \equiv R_\star L$ is ρ_R-covariant.

$$H_{\rho_R} = \int_{\Sigma_R} J_{\rho_R}$$ is a non-trivial bulk Hamiltonian: changes values of physical clock!

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Frame Changes and Frame Reorientations

Covariance

Changes of Frames[1]

- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant field-dependent perspective change.

Frame covariance: Frame changes change the observables, but not the field configuration.

- **Relational atlas**: Frames are $(\mathcal{M}, \mathcal{F})$-local. Frame changes define transition maps for meta-atlas!

Symmetries

Frame Reorientations

- **Reorientations:**[1] Change configuration of $R[\varphi]$ only: φ-indep. $R[\varphi] \to \tilde{f} \circ R[\varphi]$, $\tilde{f} \in \mathrm{Diff}(\mathcal{O}_R)$.

Physical transformations: Reorientations change the observables by changing the R-configuration.

Time evolution

- Reorientations generated by φ-dep. timelike $\rho_R[\varphi] \in \mathfrak{X}(\mathcal{O}_R)$, and $L_R \equiv R_\star L$ is ρ_R-covariant.

$$\not{\delta} H_{\rho_R} = \int_{\Sigma_R} (\delta J_{\rho_R} - J_{\delta \rho_R}) \quad \text{may define open system time-evolution}$$

[1] Goeller, Höhn, Kirklin 2022; [2] Bergmann 1961; Rovelli, 1991; [3] Giddings 2006-2016; Dittrich 2006-2007; Giesel, T. Thiemann 2015; Rejzner 2017...

Diffeos

Gauge-Invariance vs Locality

- **Gravity gauge-symmetries**: (small) active diffeos: $\varphi \sim f_\star\varphi$, $f \in H[\varphi] \subset \mathrm{Diff}(\mathcal{M})$.
- **Gauge-invariant observables** $O[\varphi] = O[f_\star\varphi]$ encode physical info but cannot be $\mathcal{M}$-local

Spacetime locality $\longrightarrow$ Relational locality!

Dynamical Frames

Relational Observables

- **Dynamical Frames for Diffeos**: φ-dependent maps $R[\varphi] : \mathcal{U}_R \to \mathcal{U}_\mathcal{O}$ such that $R[f_\star\varphi] = R \circ f^{-1}$.

Relational observables:[1,2] $\mathcal{M}$-local $A[\varphi] \longrightarrow \mathcal{O}_R$-local gauge-inv. $O_{A|R}[\varphi] = (R[\varphi]_\star)A[\varphi]$.

✓ E.g.: Scalar fields, Brown-Kuchař dust, geodesic frames. . .

Physical Transf.

Frame Changes and Frames Reorientations

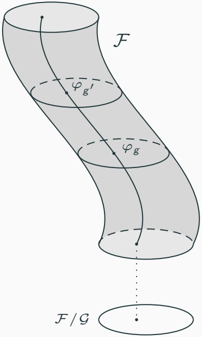
- Frames (R_1, R_2) with relational spacetimes $(\mathcal{O}_1, \mathcal{O}_2)$: $U_{1\to 2}[\varphi] = (R_1[\varphi])_\star R_2[\varphi] : \mathcal{O}_1 \to \mathcal{O}_2$.
- **Frame changes**: $U_{1\to 2}[\varphi]$ gauge-invariant φ-dependent perspective change $O_{|R_2} = (U_{1\to 2})_\star O_{|R_1}$.
- **Frame Reorientations**: Physical transformations induced by changes in R-configurations.

Spacetime covariance/symmetries $\longrightarrow$ Relational covariance/symmetries!

Setting

Field Space For Gauge Theories. . . [2]

- **Field space $\mathcal{F}$**: is a manifold constructed out of the set of field histories.
- $\mathcal{F}$ can be coordinatized by field components $\varphi^i(x) \equiv \varphi^i$.
- $\mathcal{G}$ decomposes $\mathcal{F}$ into gauge orbits: $\mathcal{F}$ is a principal fiber bundle.



[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; DeWitt 1964; Vilkovisky, Christensen 1984; . . .

The Relational Path-Integral (I)

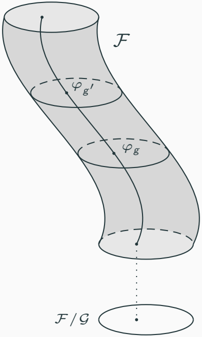
Setting

Field Space For Gauge Theories. . . [2]

- **Field space $\mathcal{F}$**: is a manifold constructed out of the set of field histories.
- $\mathcal{F}$ can be coordinatized by field components $\varphi^i(x) \equiv \varphi^i$.
- $\mathcal{G}$ decomposes $\mathcal{F}$ into gauge orbits: $\mathcal{F}$ is a principal fiber bundle.

. . . and its Geometry[2]

- Geometry of $\mathcal{F}$: defined by a gauge-invariant $\mathcal{F}$-metric $\mathfrak{G}_{ij}$.
- $\mathfrak{G}_{ij}$ and gauge generators induce a gauge orbits metric $\gamma_{\alpha\beta}$.



[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; DeWitt 1964; Vilkovisky, Christensen 1984; . . .

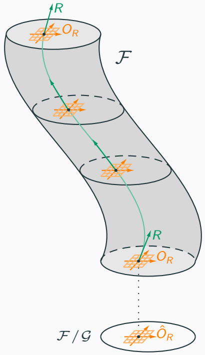
Setting

Field Space For Gauge Theories. . . [2]

- **Field space** $\mathcal{F}$: is a manifold constructed out of the set of field histories.
- $\mathcal{F}$ can be coordinatized by field components $\varphi^i(x) \equiv \varphi^i$.
- $\mathcal{G}$ decomposes $\mathcal{F}$ into gauge orbits: $\mathcal{F}$ is a principal fiber bundle.

. . . and its Geometry[2]

- Geometry of $\mathcal{F}$: defined by a gauge-invariant $\mathcal{F}$-metric $\mathfrak{G}_{ij}$.
- $\mathfrak{G}_{ij}$ and gauge generators induce a gauge orbits metric $\gamma_{\alpha\beta}$.



Dyn. Frames

Fiber Adapted Coordinates[1]

Relational field coordinates: (R, O_R) are fiber adapted coordinates for $\mathcal{F}$.

- Introduced by DeWitt; reluctantly abandoned as gauge-invariant coordinates are $\mathcal{M}$-non-local.
- Revived by relational update on locality and covariance!

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; DeWitt 1964; Vilkovisky, Christensen 1984; . . .

The Relational Path-Integral (I)

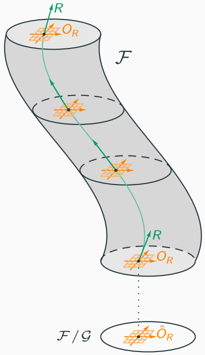
Setting

Field Space For Gauge Theories... [2]

- **Field space** $\mathcal{F}$: is a manifold constructed out of the set of field histories.
- $\mathcal{F}$ can be coordinatized by field components $\varphi^i(x) \equiv \varphi^i$.
- $\mathcal{G}$ decomposes $\mathcal{F}$ into gauge orbits: $\mathcal{F}$ is a principal fiber bundle.

...and its Geometry[2]

- Geometry of $\mathcal{F}$: defined by a gauge-invariant $\mathcal{F}$-metric $\mathfrak{G}_{ij}$.
- $\mathfrak{G}_{ij}$ and gauge generators induce a gauge orbits metric $\gamma_{\alpha\beta}$.



Dyn. Frames

Fiber Adapted Coordinates[1]

Relational field coordinates: (R, O_R) are fiber adapted coordinates for $\mathcal{F}$.

- **Frame changes**: Are passive coordinate changes on $\mathcal{F}$: $(R, O_R) \to (R', O_{R'})$.
- **Reorientations**: Active change of R-configuration, which changes O_R.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; DeWitt 1964; Vilkovisky, Christensen 1984; ...

The Relational Path-Integral (I)

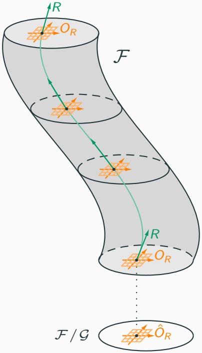
Setting

Field Space For Gauge Theories. . . [2]

- **Field space** $\mathcal{F}$: is a manifold constructed out of the set of field histories.
- $\mathcal{F}$ can be coordinatized by field components $\varphi^i(x) \equiv \varphi^i$.
- $\mathcal{G}$ decomposes $\mathcal{F}$ into gauge orbits: $\mathcal{F}$ is a principal fiber bundle.

. . . and its Geometry[2]

- Geometry of $\mathcal{F}$: defined by a gauge-invariant $\mathcal{F}$-metric $\mathfrak{G}_{ij}$.
- $\mathfrak{G}_{ij}$ and gauge generators induce a gauge orbits metric $\gamma_{\alpha\beta}$.



Dyn. Frames

Fiber Adapted Coordinates[1]

Relational field coordinates: induce coordinates $\hat{O}_R$ on $\mathcal{F}/\mathcal{G}$ by projection.

- **Frame changes**: Are passive coordinate changes on $\mathcal{F}/\mathcal{G}$: $\hat{O}_R \to \hat{O}_{R'}$
- **Reorientations**: Active change of R-configuration, which changes $\hat{O}_R$.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; DeWitt 1964; Vilkovisky, Christensen 1984; . . .

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral...[1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\,\det\gamma]^{1/2}[\hat{O}_R]$.

  Relational observables projected to base

  Metric in the O_R directions projected to the base

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992...

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral...[1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\,\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \left\{\det\hat{\mathfrak{G}}\,\det\gamma\right\}^{1/2}[\hat{O}_R]\exp\{iS_R[\hat{O}_R]\}\,.$$

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992...

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral... [1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_{R'} \left\{\det\hat{\mathfrak{G}}\det\gamma\right\}^{1/2}[\hat{O}_{R'}]\exp\{iS_{R'}[\hat{O}_{R'}]\}\,.$$

...and its Properties[1]

✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992...

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral. . . [1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int \mathcal{D}R\delta[R,\bar{R}] \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \left\{\det\hat{\mathfrak{G}}\det\gamma\right\}^{1/2}[\hat{O}_R]\exp\{iS_R[\hat{O}_R]\}\,.$$

. . . and its Properties[1]

- ✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.
- Quantum frame reduction: Physical gauge-fixing $R = \tilde{R}$ ($\tilde{R}$ fixed) leads to the same PN Z.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992. . .

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral. . . [1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\,\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}} \mathcal{D}\varphi\sqrt{\det\mathfrak{G}}\,\delta[R,\tilde{R}]\det\Delta_R\exp\{iS[\varphi]\}\,.$$

. . . and its Properties[1]

- ✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.
- Quantum frame reduction: Physical gauge-fixing $R = \tilde{R}$ ($\tilde{R}$ fixed) leads to the same PN Z.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992. . .

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral...[1]

- Measure on $\mathcal{F}/\mathcal{G}$:[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det \hat{\mathfrak{G}} \det \gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}} \mathcal{D}\varphi \sqrt{\det \mathfrak{G}}\, \delta[\chi, 0] \det \Delta_R \exp\{iS[\varphi]\} \,.$$

...and its Properties[1]

- ✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.
- Quantum frame reduction: Physical gauge-fixing $R = \tilde{R}$ ($\tilde{R}$ fixed) leads to the same PN Z.
- FP quantization: $\chi[\varphi] = R - \tilde{R} = 0$ equivalent to quantum frame reduction and PN.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992...

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral. . .[1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\,\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \left\{\det\hat{\mathfrak{G}}\right\}^{1/2}[\hat{O}_R]\exp\{iS_R[\hat{O}_R]\}\,.$$

✓ Quantum Frame Fields: R are field-theoretic QRFs!

. . . and its Properties[1]

✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.

- Quantum frame reduction: Physical gauge-fixing $R = \tilde{R}$ ($\tilde{R}$ fixed) leads to the same PN Z.
- FP quantization: $\chi[\varphi] = R - \tilde{R} = 0$ equivalent to quantum frame reduction and PN.
- Classical frame reduction[3]: lack of gauge-orbit contribution ($\det\gamma$) in Z; relevant for anomalies.

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992. . .

The Relational Path-Integral (II)

Quantum Reference Fields

The Perspective Neutral Relational Path-Integral... [1]

- **Measure on $\mathcal{F}/\mathcal{G}$:**[2] Induced by $\mathfrak{G}$, $\mathcal{D}\hat{O}_R\mu[\hat{O}_R] = \mathcal{D}\hat{O}_R[\det\hat{\mathfrak{G}}\det\gamma]^{1/2}[\hat{O}_R]$.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \left\{\det\hat{\mathfrak{G}}\det\gamma\right\}^{1/2}[\hat{O}_R]\exp\{iS_R[\hat{O}_R]\}\,.$$

- ✓ Quantum Frame Fields: R are field-theoretic QRFs!

...and its Properties[1]

- ✓ Perspective neutral: Z invariant under frame changes as they amount to $(R, O_R) \to (R', O_{R'})$.
- Quantum frame reduction: Physical gauge-fixing $R = \tilde{R}$ ($\tilde{R}$ fixed) leads to the same PN Z.
- FP quantization: $\chi[\varphi] = R - \tilde{R} = 0$ equivalent to quantum frame reduction and PN.
- Classical frame reduction[3]: lack of gauge-orbit contribution ($\det\gamma$) in Z; relevant for anomalies.
- ✓ Regularization: Lattice/regulators on orientation space do not break spacetime background indep.!

[1] Aguilar, Ferrero, Höhn, LM (to appear); [2] Parker, Toms, 2009: DeWitt, 2003; Vilkovisky, Christensen 1984; [3] Kunstatter 1992...

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- Why are QRFs so important for quantum gravity?
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- What are quantum reference frames for gravity?
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.
- How to define a relational gravitational path-integral? How to make sense of it mathematically?
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- How is it related to the reduced and gauge-fixed (FP) path integrals?
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- How does it transform under QRF changes?

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Part I

QRF and Mechanical Models

- QRFs are dynamical quantities that locally parametrize gauge orbits.
- QRFs allow us to build relational observables capturing all gauge-invariant quantum info.
- QRFs provide us with fiber-adapted field space coordinates: general geometric construction!

Part II

The Relational Gravitational Path Integral

- The only observables in classical and quantum gravity are relational, defined wrt. a QRF.
- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

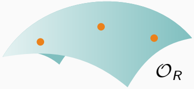
Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking states defined?
- How do ground sectors look like from different QRF perspectives?

Sharp and Fuzzy Correlators

$$\left\langle \hat{O}_{A_1|R}(o_1)\cdots\hat{O}_{A_n|R}(o_n)\right\rangle_R = \frac{1}{Z}\int \mathcal{D}\hat{O}_R\,\mu_R[\hat{O}_R]\,\hat{O}_{A_1|R}(o_1)\cdots\hat{O}_{A_n|R}(o_n)\,e^{iS_R[\hat{O}_R]}$$



Aguilar, Ferrero, Höhn, LM 2607.21463.

Sharp and Fuzzy Correlators

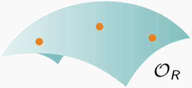
$$\left\langle \hat{O}_{A_1|R}(o_1)\cdots \hat{O}_{A_n|R}(o_n)\right\rangle_R = \frac{1}{Z}\int \mathcal{D}\hat{O}_R\, \mu_R[\hat{O}_R]\, \hat{O}_{A_1|R}(o_1)\cdots \hat{O}_{A_n|R}(o_n)\, e^{iS_R[\hat{O}_R]}$$

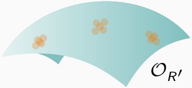
$= U_{R\to R'}[\hat{\varphi}](o_i)$: field-dependent point

$$\hat{O}_{A_i|R}(o_i) = \hat{O}_{\tilde{A}_i|R'}(o_i'[\hat{\varphi}])$$

$= \hat{O}_{A|R}\circ U_{R'\to R}$: scalarized observables

$$\left\langle \hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots \hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\right\rangle_{R'} = \frac{1}{Z}\int \mathcal{D}\hat{O}_{R'}\, \mu_{R'}[\hat{O}_{R'}]\, \hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots \hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\, e^{iS_{R'}[\hat{O}_{R'}]}$$





Aguilar, Ferrero, Höhn, LM 2607.21463.

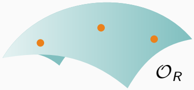
Quantum Correlators

Sharp and Fuzzy Correlators

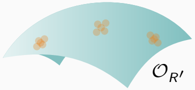
$$\left\langle \hat{O}_{A_1|R}(o_1)\cdots\hat{O}_{A_n|R}(o_n)\right\rangle_R = \frac{1}{Z}\int \mathcal{D}\hat{O}_R\,\mu_R[\hat{O}_R]\,\hat{O}_{A_1|R}(o_1)\cdots\hat{O}_{A_n|R}(o_n)\,e^{iS_R[\hat{O}_R]}$$

$$\Big\downarrow\ \hat{O}_{A_i|R}(o_i) = \hat{O}_{\tilde{A}_i|R'}(o_i'[\hat{\varphi}])$$

$$\left\langle \hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots\hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\right\rangle_{R'} = \frac{1}{Z}\int \mathcal{D}\hat{O}_{R'}\,\mu_{R'}[\hat{O}_{R'}]\,\hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots\hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\,e^{iS_{R'}[\hat{O}_{R'}]}$$



Sharp correlators in R-perspective become fuzzy in R'-perspective!



Aguilar, Ferrero, Höhn, LM 2607.21463.

Relational Correlators

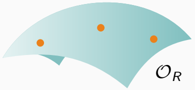
Quantum Correlators

Sharp and Fuzzy Correlators

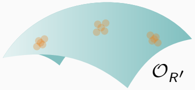
$$\left\langle \hat{O}_{A_1|R}(o_1)\cdots\hat{O}_{A_n|R}(o_n)\right\rangle_R = \frac{(-i)^n}{Z_R}\,\frac{\delta^n Z_R[J_R]}{\delta J_R(o_1)\cdots\delta J_R(o_n)}\bigg|_{J_R=0}$$

$$\Big\downarrow\ \hat{O}_{A_i|R}(o_i) = \hat{O}_{\tilde{A}_i|R'}(o_i'[\hat{\varphi}])$$

$$\left\langle \hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots\hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\right\rangle_{R'} = \frac{1}{Z}\int\mathcal{D}\hat{O}_{R'}\,\mu_{R'}[\hat{O}_{R'}]\,\hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots\hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\,e^{iS_{R'}[\hat{O}_{R'}]}$$



Sharp correlators in R-perspective become fuzzy in R'-perspective!



Sources

Generating Functionals

$$Z_R[J_R] \equiv e^{iW_R[J_R]} = \int_{\mathcal{F}/\mathcal{G}}\mathcal{D}\hat{O}_R\,\mu_R[\hat{O}_R]\,e^{iS_R[\hat{O}_R]+i(J_R)_A\hat{O}_R^A}$$

- Sources J_R are background fields on $\mathcal{O}_R$, thus manifestly gauge-invariant.

Aguilar, Ferrero, Höhn, LM 2607.21463.

Relational Correlators

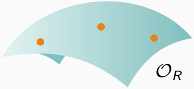
Quantum Correlators

Sharp and Fuzzy Correlators

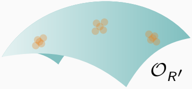
$$\left\langle \hat{O}_{A_1|R}(o_1)\cdots \hat{O}_{A_n|R}(o_n)\right\rangle_R = \frac{(-i)^n}{Z_R}\,\frac{\delta^n Z_R[J_R]}{\delta J_R(o_1)\cdots \delta J_R(o_n)}\bigg|_{J_R=0}$$

$$\hat{O}_{A_i|R}(o_i) = \hat{O}_{\tilde{A}_i|R'}(o_i'[\hat{\varphi}])$$

$$\left\langle \hat{O}_{\tilde{A}_1|R'}(o_1'[\hat{\varphi}])\cdots \hat{O}_{\tilde{A}_n|R'}(o_n'[\hat{\varphi}])\right\rangle_{R'} \neq \frac{(-i)^n}{Z_{R'}}\,\frac{\delta^n Z_{R'}[J_{R'}]}{\delta J_{R'}(o_1')\cdots \delta J_{R'}(o_1')}\bigg|_{J_{R'}=0}$$



Sharp correlators in R-perspective become fuzzy in R'-perspective!



Sources

Generating Functionals

$$Z_R[J_R] \equiv e^{iW_R[J_R]} = \int_{\mathcal{F}/\mathcal{G}} \mathcal{D}\hat{O}_R\, \mu_R[\hat{O}_R]\, e^{iS_R[\hat{O}_R]+i(J_R)_A \hat{O}_R^A}$$

- Sources J_R are background fields on $\mathcal{O}_R$, thus manifestly gauge-invariant.
- $Z_R[J_R]$ cannot be frame covariant!

Aguilar, Ferrero, Höhn, LM 2607.21463.

The Relational Effective Action

Definition

Relational effective action[1]

$$e^{i\Gamma_R[O_R]} = \int \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R] - i\Gamma_{R,A}(\hat{O}^A_R - O^A_R)}$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 [2] Wegner 1974, Weinberg, 1979.

The Relational Effective Action

Definition

Relational effective action[1]

$$e^{i\Gamma_R[O_R]} = \int \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R] - i\Gamma_{R,A}(\hat{O}^A_R - O^A_R)}$$

✓ Manifestly gauge- and background-independent off shell and at the non-perturbative level.

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 [2] Wegner 1974, Weinberg, 1979.

The Relational Effective Action

Definition

Relational effective action[1]

$$e^{i\Gamma_R[O_R]} = \int \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R] - i\Gamma_{R,A}(\hat{O}_R^A - O_R^A)}$$

✓ Manifestly gauge- and background-independent off shell and at the non-perturbative level.

QRF Changes

QRF covariance of on-shell physics[1]

- R-quantum-shell: $(J_R)_A = \Gamma_{R,A} = 0$. Perspective neutral quantum shell $J_R = 0$ for all R.
- Frame changes do not fix background J_R changes, but covariance does: $J_{R'}[J_R = 0] = 0$!

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 [2] Wegner 1974, Weinberg, 1979.

The Relational Effective Action

Definition

Relational effective action[1]

$$e^{i\Gamma_R[O_R]} = \int \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R] - i\Gamma_{R,A}(\hat{O}_R^A - O_R^A)}$$

✓ Manifestly gauge- and background-independent off shell and at the non-perturbative level.

QRF Changes

QRF covariance of on-shell physics[1]

- R-quantum-shell: $(J_R)_A = \Gamma_{R,A} = 0$. Perspective neutral quantum shell $J_R = 0$ for all R.
- Frame changes do not fix background J_R changes, but covariance does: $J_{R'}[J_R = 0] = 0$!

Infinitesimal QRF changes[1]

- Frame-Nielsen identity for one-parameter family of frames R_λ, $\lambda \in \mathbb{R}$:

$$\delta_\lambda(\Gamma_\lambda[O_\lambda]) = \Gamma_{\lambda,A}[O_\lambda](\delta_\lambda O_\lambda^A - \langle \delta_\lambda \hat{O}_\lambda^A \rangle_{J_\lambda})$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 [2] Wegner 1974, Weinberg, 1979.

The Relational Effective Action

Definition

Relational effective action[1]

$$e^{i\Gamma_R[O_R]} = \int \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R] - i\Gamma_{R,A}(\hat{O}^A_R - O^A_R)}$$

- ✓ Manifestly gauge- and background-independent off shell and at the non-perturbative level.
- ✓ On-shell covariance of the effective action: On the quantum shell, $\Gamma_{R,A} = 0$, $\Gamma_R \approx \Gamma_{R'}$.

QRF Changes

QRF covariance of on-shell physics[1]

- R-quantum-shell: $(J_R)_A = \Gamma_{R,A} = 0$. Perspective neutral quantum shell $J_R = 0$ for all R.
- Frame changes do not fix background J_R changes, but covariance does: $J_{R'}[J_R = 0] = 0$!

Infinitesimal QRF changes[1]

- Frame-Nielsen identity for one-parameter family of frames R_λ, $\lambda \in \mathbb{R}$:

$$\delta_\lambda(\Gamma_\lambda[O_\lambda]) = \Gamma_{\lambda,A}[O_\lambda](\delta_\lambda O^A_\lambda - \langle \delta_\lambda \hat{O}^A_\lambda \rangle_{J_\lambda})$$

- Frame changes = redundant transf.[2]; associated coupling λ is inessential[2] (important for RG).

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 [2] Wegner 1974, Weinberg, 1979.

R-data in R perspective

$$K_R[\bar{O}^i_R, \Sigma^i_R; \bar{O}^f_R, \Sigma^f_R] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$

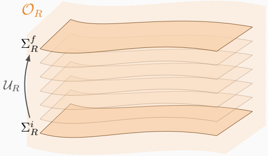
[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Höhn, Smith, Lock 1912.00033.

Time Evolution

R-data in R perspective

$$K_R[\bar{O}^i_R, \Sigma^i_R; \bar{O}^f_R, \Sigma^f_R] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$

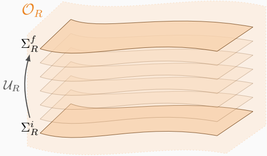
- Take $L_R = R_\star L$ covariant wrt. field-indep. timelike $\rho \in \mathfrak{X}(\mathcal{O}_R)$.
- Then H_{ρ_R} generates integrable time-evolution.
- K_R defines unitary evolution operator $\hat{U}_R(\Sigma^i_R, \Sigma^f_R)$.



Time Evolution

R-data in R perspective

$$K_R[\bar{O}_R^i, \Sigma_R^i; \bar{O}_R^f, \Sigma_R^f] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$

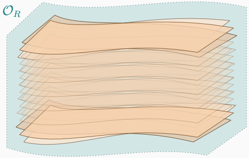


- Take $L_R = R_\star L$ covariant wrt. field-indep. timelike $\rho \in \mathfrak{X}(\mathcal{O}_R)$.
- Then H_{ρ_R} generates integrable time-evolution.
- K_R defines unitary evolution operator $\hat{U}_R(\Sigma_R^i, \Sigma_R^f)$.

QRF Changes

R-data in R'-perspective

$$K_R[\bar{O}_R^i, \Sigma_R^i; \bar{O}_R^f, \Sigma_R^f] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$



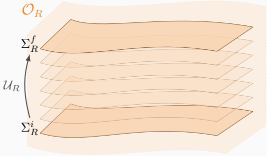
- ρ is R-reor., so can only change $O_{R|R'}$; not R'-evolution!
- $\rho'[\hat{\varphi}] \equiv (U_{R\to R'}[\hat{\varphi}])_\star \rho \in \mathfrak{X}(\mathcal{O}_{R'})$ is field-dependent!
- No closed system (unitary) dynamics in R'-perspective![2]

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Höhn, Smith, Lock 1912.00033.

Time Evolution

R-data in R perspective

$$K_R[\bar{O}^i_R, \Sigma^i_R; \bar{O}^f_R, \Sigma^f_R] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$

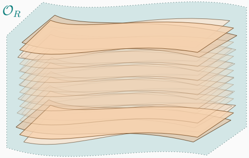


- Take $L_R = R_\star L$ covariant wrt. field-indep. timelike $\rho \in \mathfrak{X}(\mathcal{O}_R)$.
- Then H_{ρ_R} generates integrable time-evolution.
- K_R defines unitary evolution operator $\hat{U}_R(\Sigma^i_R, \Sigma^f_R)$.

QRF Changes

R-data in R'-perspective

$$K_R[\bar{O}^i_R, \Sigma^i_R; \bar{O}^f_R, \Sigma^f_R] = \int_{(\mathcal{F}/\mathcal{G})^{i,f}} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] \, e^{iS_R[\hat{O}_R]}$$



- ρ is R-reor., so can only change $O_{R|R'}$; not R'-evolution!
- $\rho'[\hat{\varphi}] \equiv (U_{R\to R'}[\hat{\varphi}])_\star \rho \in \mathfrak{X}(\mathcal{O}_{R'})$ is field-dependent!
- No closed system (unitary) dynamics in R'-perspective![2]

Sharp time evolution may appear fuzzy from another QRF's perspective!

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Höhn, Smith, Lock 1912.00033.

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.

$$\Psi_{\mathrm{HH}}[\bar{\varphi}|_{\Sigma}] = \int_{\mathcal{C}[\Sigma]} \mathcal{D}\varphi \sqrt{\det \mathfrak{G}}\, e^{-\mathcal{S}}$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian ⟶ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.

$$\Psi_{\mathrm{HH},R}[\bar{O}_R|_{\Sigma_R}] = \int_{\mathcal{C}[\Sigma_R]} \mathcal{D}\hat{O}_R \, \mu_R[\hat{O}_R] e^{-\mathcal{S}_R}$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

Definition

Relational Effective action

. . . but on $\mathcal{O}_R$ we do have a relational Hamiltonian (under certain conditions)!

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

Definition

Relational Effective action

... but on $\mathcal{O}_R$ we do have a relational Hamiltonian (under certain conditions)!

$$\begin{tikzcd} \hat{P}_R \arrow[d] & \hat{U}_R \arrow[l, squiggly, dashed, "\text{Eucl. past/future}"'] \\ \hat{\mathcal{H}}_{0,R} & \end{tikzcd}$$

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

Definition

Relational Effective action

...but on $\mathcal{O}_R$ we do have a relational Hamiltonian (under certain conditions)!

$$\begin{tikzcd} \hat{U}_{R'} \arrow[r, squiggly, "\text{Eucl. past/future}"] & \hat{P}_{R'} \arrow[d] \\ & \hat{\mathcal{H}}_{0,R'} \end{tikzcd} \qquad \begin{tikzcd} \hat{P}_{R} \arrow[d] & \hat{U}_{R} \arrow[l, squiggly, "\text{Eucl. past/future}"'] \\ \hat{\mathcal{H}}_{0,R} & \end{tikzcd}$$

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

Definition

Relational Effective action

... but on $\mathcal{O}_R$ we do have a relational Hamiltonian (under certain conditions)!

$$\begin{tikzcd}
\hat{U}_{R'} \arrow[r, squiggly, "\text{Eucl. past/future}"] & \hat{P}_{R'} \arrow[rr, "\text{Eucludean cont. of}", "\text{frame change map}"'] \arrow[d] & & \hat{P}_R \arrow[d] & \hat{U}_R \arrow[l, squiggly, "\text{Eucl. past/future}"'] \\
& \hat{\mathcal{H}}_{0,R'} & \text{Physically distinct!} & \hat{\mathcal{H}}_{0,R} &
\end{tikzcd}$$

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

Ground Sectors

No-Boundary

Hartle-Hawking

Absence of Hamiltonian $\longrightarrow$ No-Boundary Proposal[2]

- HH: compact Euclidean geometries in the past of Σ.
- Relational HH: same but with $\mathcal{O}_R$ and Σ_R.
- Different states as $\Sigma[\hat{\varphi}] = R^{-1}[\hat{\varphi}](\Sigma_R)$.

$$|\Psi_{\mathrm{HH},R}[\Sigma_R]\rangle \neq |\Psi_{\mathrm{HH}}[\Sigma]\rangle$$

Definition

Relational Effective action

…but on $\mathcal{O}_R$ we do have a relational Hamiltonian (under certain conditions)!

$$\begin{tikzcd}
\hat{U}_{R'} \arrow[r, squiggly, "\text{Eucl. past/future}"] & \hat{P}_{R'} \arrow[r, "\text{Eucludean cont. of}", "\text{frame change map}"'] \arrow[d] & \hat{P}_R \arrow[d] & \hat{U}_R \arrow[l, squiggly, "\text{Eucl. past/future}"'] \\
 & \hat{\mathcal{H}}_{0,R'} \arrow[r, phantom, "\text{Physically distinct!}"] & \hat{\mathcal{H}}_{0,R} &
\end{tikzcd}$$

Relational Unruh Effect[3]: ground sectors appear excited from another frame's perspective!

[1] Aguilar, Ferrero, Höhn, LM 2607.21463 and to appear; [2] Hartle, Hawking 1983; [3] Höhn, Russo, Smith 2308.12912.

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- How do local relational correlators look like in different frames?
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking (HH) states defined?
- How do ground sectors/HH states look like from different QRF perspectives?

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- What generating functionals should be considered? How to build effective actions?
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking (HH) states defined?
- How do ground sectors/HH states look like from different QRF perspectives?

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- Is quantum gravitational physics covariant under QRF changes?
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking (HH) states defined?
- How do ground sectors/HH states look like from different QRF perspectives?

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- On-shell frame covariance: Effective action and quantum EoM covariant under QRF changes!
- How do transition amplitudes look like in different frames?
- How are relational ground sectors and Hartle-Hawking (HH) states defined?
- How do ground sectors/HH states look like from different QRF perspectives?

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- On-shell frame covariance: Effective action and quantum EoM covariant under QRF changes!
- Relativity of time evolution: Sharp time evolution may appear fuzzy for different QRFs!
- How are relational ground sectors and Hartle-Hawking (HH) states defined?
- How do ground sectors/HH states look like from different QRF perspectives?

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- On-shell frame covariance: Effective action and quantum EoM covariant under QRF changes!
- Relativity of time evolution: Sharp time evolution may appear fuzzy for different QRFs!
- R-ground sectors via Euclidean relational PI; R-HH defined on relational spacetime.
- R-Unruh Effect R-ground sectors may appear excited for another QRF! R-HH for different QRFs!

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- On-shell frame covariance: Effective action and quantum EoM covariant under QRF changes!
- Relativity of time evolution: Sharp time evolution may appear fuzzy for different QRFs!
- R-ground sectors via Euclidean relational PI; R-HH defined on relational spacetime.
- R-Unruh Effect R-ground sectors may appear excited for another QRF! R-HH for different QRFs!

Coming Up

Relational RG

- How to coarse-grain in background indep. theories?
- Do QRFs define a natural notion of regulator?
- Role of QRF covariance in exact RG flows?
- How do truncations affect frame covariance?
- ...

Part II

The Relational Gravitational Path Integral

- Naturally defined on $\mathcal{F}$ as dynamical frames; induce relationally fiber adapted coordinates.

$$Z = \int \mathcal{D}\hat{O}_R \{\det \hat{\mathfrak{G}} \det \gamma\}^{1/2}[\hat{O}_R] e^{iS_R[\hat{O}_R]}$$

- Naturally regularized in orientation space: no violation of background independence!
- Different from reduced path integral ($\det \gamma$), equivalent to FP path integral.
- Perspective Neutral: manifestly invariant under QRF changes.

Part III

QRF covariance and Physical Consequences

- Event relativity: Sharp correlators may look fuzzy for different QRFs!
- Sources as background field on relational spacetime. Generating functionals not frame covariant!
- On-shell frame covariance: Effective action and quantum EoM covariant under QRF changes!
- Relativity of time evolution: Sharp time evolution may appear fuzzy for different QRFs!
- R-ground sectors via Euclidean relational PI; R-HH defined on relational spacetime.
- R-Unruh Effect R-ground sectors may appear excited for another QRF! R-HH for different QRFs!

Coming Up

Relational RG

- How to coarse-grain in background indep. theories?
- Do QRFs define a natural notion of regulator?
- Role of QRF covariance in exact RG flows?
- How do truncations affect frame covariance?
- ...

Beyond Formalism:
Quantitative Predictions

- Concrete computations: cosmology
- Open systems in QG
- QRF phenomenology